Tomographic Reconstruction of Dynamic Objects

Yonggang Shi, W.Clem Karl

Multidimensional Signal Processing Lab
Electrical and Computer Engineering Department
Boston University

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Talk Outlines

- Introduction
  - Background
  - Previous works of object-based dynamic tomography
  - Curve evolution and level set methods
- Our approach
  - Geometric object-based scene modeling
  - Shape dynamics for temporal boundary smoothness
  - Variational reconstruction with curve evolution and level set methods
- Extensions
  - Learning shape dynamics & Shape matching

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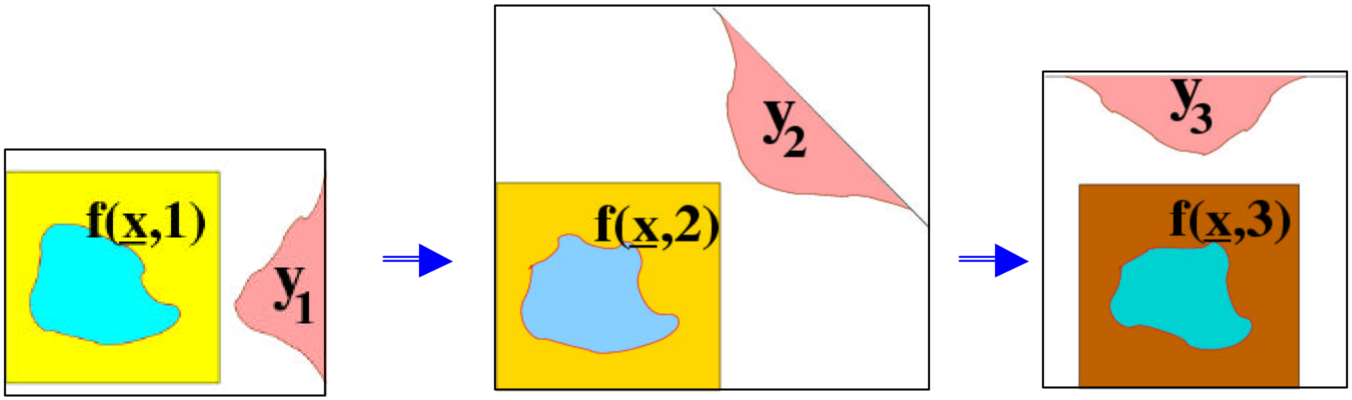
Introduction

Scenario of Dynamic Tomography



- Time varying *f*(x,*k*), 2-D or 3-D
- Noisy tomographic projection data y_k , for example, line integration
- Limited view angles at each time: 1~3
- Observation angles change over time
- **Goal:** reconstruct $\boldsymbol{f}$ from $\boldsymbol{y}$

Motivating Applications

- Nuclear medicine
  - PET(Positron Emission Tomography)
  - SPECT (Single Photon Emission Computed Tomography)
  - First pass and equilibrium blood pool imaging.
  - Myocardium perfusion imaging.
- Imaging of explosive events

A 3-head SPECT

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Interest in Problems Where:

- Characteristics
  - Scenes composed of few discrete “objects”
  - Interest in object localization/characterization
  - Simple “textures” or less interest in texture
- Challenges
  - Data are sparse and noisy
  - Tomographic operator is ill-posed
  - High dimension: 2-D or 3-D image sequence

Challenging Inverse Problem!

Previous Works: Object-based Dynamic Tomography

- P.C.Chiao, W.L.Rogers, N.H.Clinthorne, J.A.Fessler, and A.O.Hero, 1994
  - Cardiac PET study with a polygonal model
  - Joint estimation left ventricle boundary and dynamic intensity parameters
  - Known topology and static boundary
  - Static tomographic scenario with multiple projections

P.C.Chiao, W.L.Rogers, N.H.Clinthorne, J.A.Fessler, and A.O.Hero, *"Model-based estimation for dynamic cardiac studies using ECT"*, *IEEE Trans Medical Imaging, 1994.*

- G.S.Cunningham, K.M.Hanson, and X.L.Battle,1998:
  - First pass blood pool imaging of the right ventricle of an artificial heart from very noisy SPECT data
  - 3-D triangulated surface model evolves over time
  - Difficult to handle topological changes
  - 24 view obtained at each time
  - No temporal modeling of object boundary

G.S.Cummingham, K.M.Hanson, and X.L.Battle, "Three-dimensional reconstruction from low-count SPECT data using deformable models",Optics Express, 1998.

- Tom Asaki and Kevin Vixie, 2002
  - Reconstructing a parameterized evolving curve
  - Single angle projection at each time
  - No topological changes and temporal modeling

Tom Asaki and Kevin R. Vixie, "Reconstruction of evolving , non-convex curves from a sequence of single angle projections", 1st SIAM Imaging Science Conference, Boston, 2002.

Curve Evolution Methods

- Snake

M.Kass, A. Witkin and D. Terzopoulos, "Snakes:active contour models", IJCV,1988.

- Edge-based Active Contours

V. Caselles and R. Kimmel and G. Sapiro, "Geodesic Active contours",IJCV, 1997.

- Region-based Active Contours

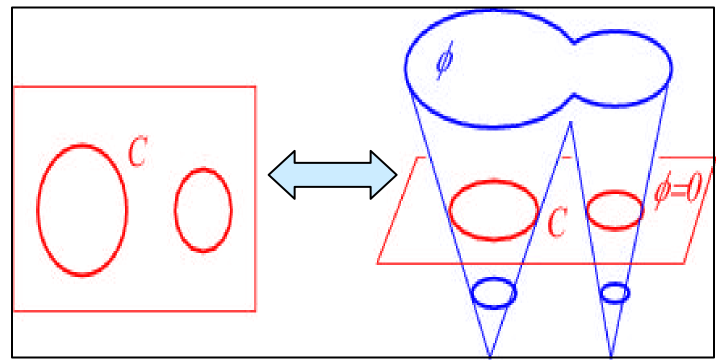
T.F. Chan and L.A.Vese, "Active contours without edges", IEEE Trans Image Processing, 2001.

A.Tsai, A.Yezzi, and A.Willsky, "Curve Evolution Implementation of the Mumford-Shah Functional for Image Segmentation, Denoising, Interpolation, and Magnification", IEEE Trans Image Processing, 2001.

Level Set Methods

- S.Osher and J.A. Sethian
  - Implicit shape representation
  - Solving PDEs to evolve curves, surfaces
  - Easy topological changes
  - Geometric quantity easy to

S.Osher and J.A.Sethian, "Fronts propagation with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulations", Journal of computational physics, 1988.



$$\frac{dC}{dt} = F\vec{N}$$

$$\frac{df}{dt} + F\left|\nabla f\right| = 0$$

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Our Approach

- Geometric object-based scene modeling
  - Continuous curves, surfaces for object boundary
  - Observation model based on geometric scene modeling
- Modeling object temporal dynamics
  - Shape dynamics for temporal boundary smoothness to improve robustness to data sparsity and noise
- Unified variational formulation
  - Curve evolution methods for joint estimation of boundaries and intensities
  - Apply level set methods to infer topological uncertainty

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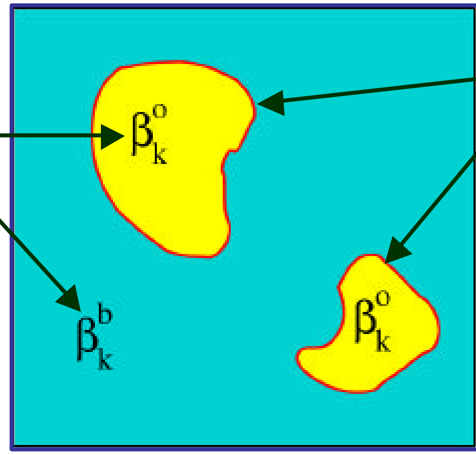
Scene Modeling

Intensity parameter: b_k



Boundary Curve: $\boldsymbol{C_k}$
Object Region: G_{C_k}
Background region: $W \backslash G_{C_k}$

Region indication function

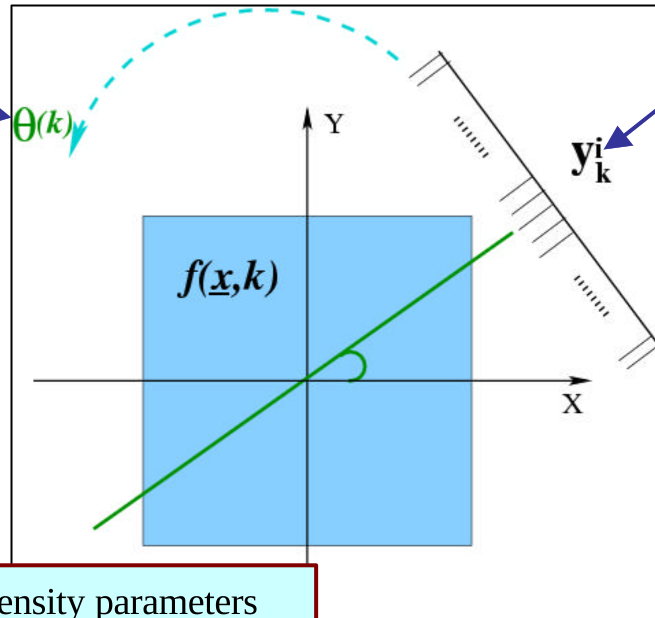
$$f(\underline{x}, k) = b_k^o r_k(\underline{x}) + b_k^b (1 - r_k(\underline{x}))$$

- Unknowns to estimate for reconstruction:
  - –Boundary sequence: $\boldsymbol{C = [C_1, C_2, \cdots, C_K]}$
  - –Intensity sequence: $b = [b_1, b_2, \cdots, b_K]$

Observation Model

Observation angle changes over time

i-th sample at time k



Intensity parameters

$$y_k^i = b_k^o \int_{\Gamma_{C_k}} H_k^i(\underline{x})\,d\underline{x} + b_k^b \int_{\Omega \setminus \Gamma_{C_k}} H_k^i(\underline{x})\,d\underline{x} + w_k^i$$

Object region

Background region

Projection kernel

Additive noise

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Vector Form of Observation Model

$$y = H\, b + w$$

Observation data vector

Observation matrix

Intensity vector

noise vector

$$H = \begin{bmatrix} H_1 & & & & \\ & \ddots & & & \\ & & H_k & & \\ & & & \ddots & \\ & & & & H_K \end{bmatrix}$$

function of C_k

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Object Dynamics I: Intensity Dynamics

- Autoregressive Model:

$$b_{k+1} = B_k b_k + u_k$$

Intensity at time $k+1$

System Matrix at time k

Intensity at time k

Gaussian Noise
$u_k = N(0, P_k)$

- Binary model for current experiments
- Need to establish correspondence for more complicated cases

Object Dynamics II: Shape Dynamics

- Shape dynamics based on affine transform:

$$C_{k+1} = A_k(C_k) + v_k$$

Boundary curve at time *k+1*

Affine transform at time *k*

Boundary curve at time *k*

Smooth variation to account for model error

- Global motion model for each point on the curve

Variational Reconstruction

- Joint estimation of (C,b) as the minimizer of an energy function:

$$\boxed{(\hat{C},\hat{b}) = \arg\min_{(C,b)} E(C,b)}$$

$$E(C,b) = \sum_k \left(\underbrace{\left[y_k - H_k f_k(C_k,b_k)\right]^2_{Q_k^{-1}}} + l\,\underbrace{\|C_k\|} + x\,\underbrace{\|b_{k+1} - B_k b_k\|^2_{P_k^{-1}}} + a\,\underbrace{W\left(C_{k+1}, A_k(C_k)\right)} \right)$$

E_d

Data fidelity term

E_s

Spatial shape smoothness prior

E_i

Intensity dynamics

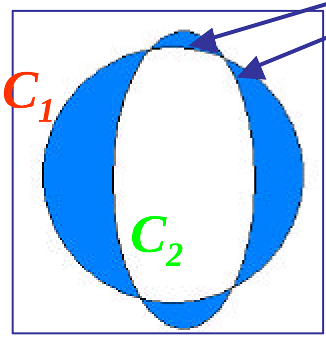
E_t

Shape dynamics: **Distance between Curves**

Distance between Curves for Shape Dynamics

- Distance between two curves:

$$W(C_1, C_2) = \int_{\Pi(C_1, C_2)} | f_{c_1}(\underline{x}) - f_{c_2}(\underline{x}) |^p \, d\underline{x}$$



Implicit representation:

$C_1 \rightarrow f_{C_1}$ negative inside

$C_2 \rightarrow f_{C_2}$ positive outside

Exponent p:

p=0 : area of $\Pi(C_1,C_2)$

p>=1: penalize parts far away

p=1 in current experiments

- **Incorporation of shape dynamics:**

$$E_t = \sum_k W(C_{k+1}, A_k(C_k))$$

Solution Approach

- "Coordinate Descent" between intensity parameters b and object boundaries C

Minimize w.r.t b : a low order quadratic optimization problem

Minimize w.r.t C_k : evolve C_k in the gradient descent direction with level set methods:

$$\boxed{\frac{dC_k}{dt} = -\nabla_{C_k} E}$$

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Solution Details I:

- First variation of data fidelity term ***E*_*d*** w.r.t ***C*_*k***:

$$\boxed{\nabla_{C_k} E_d = \left\langle -Q^{-1}(y - Y), \left[\nabla_{C_k} Y_1^1, \cdots, \nabla_{C_k} Y_K^M\right]^T \right\rangle}$$

Covariance matrix

Clean Data
$Y = H\,b$

$$\nabla_{C_k} Y_j^i = \begin{cases} (b_k^o - b_k^b) H_k^i (C_k) \vec{N}_{C_k} & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}$$

- First variation of spatial smooth prior ***E*_*s*** w.r.t ***C*_*k***:

$$\boxed{\nabla_{C_k} E_s = k\,\vec{N}_{C_k}}$$

Curvature

Normal

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Solution Details II:

- First variation of shape dynamics term E_t w.r.t C_k when p = 1:

$$\nabla_{C_k} E_t = (\underbrace{f_{\hat{C}_k}(C_k)}_{\text{Temporal smoothness to previous curve}} + \underbrace{f_{C_{k+1}}(\hat{C}_{k+1})\left|L_k\right|}_{\text{Temporal smoothness to next curve}})\vec{N}_{C_k}$$

Temporal smoothness to previous curve

Temporal smoothness to next curve

- Notations: $A_k(\underline{x}) = L_k \underline{x} + b_k$

$$\hat{C}_k = A_{k-1}(C_{k-1}), \hat{C}_{k+1} = A_k(C_k), \hat{C}_k \rightarrow f_{\hat{C}_k}, C_{k+1} \rightarrow f_{C_{k+1}}$$

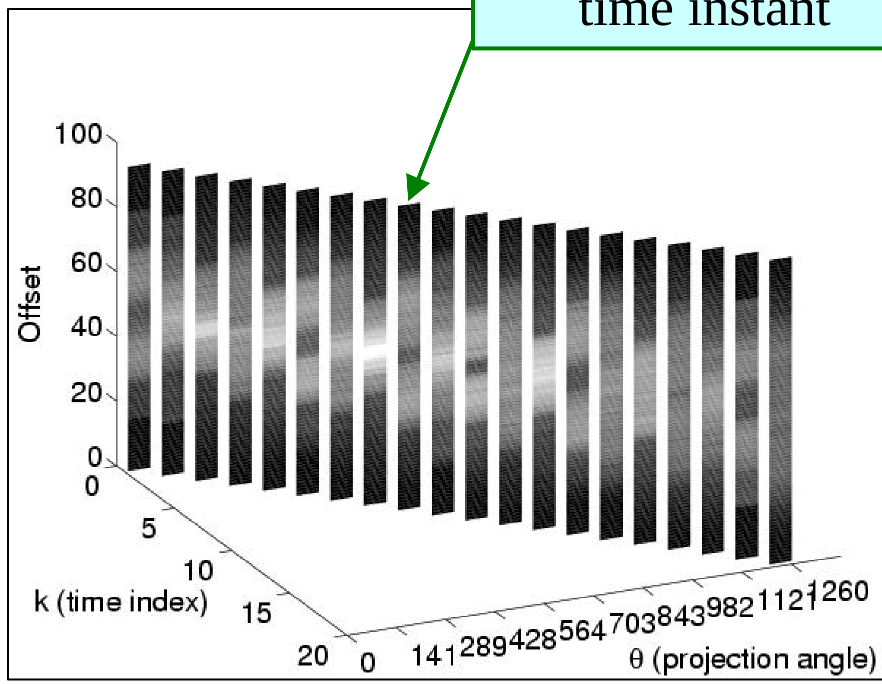
Experiment I: 2D + Time

Experiment setup:

- Observation kernel: line integration along a single angle at each time instant
- View angle changes over time
- Dynamic models A_k, B_k : identity transform
- Gaussian noise 27dB
- Sequence size : 64*64*20

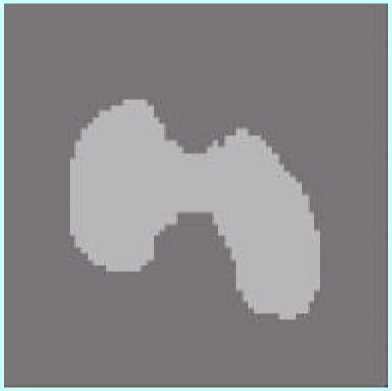
Observation data:

Observation at one time instant

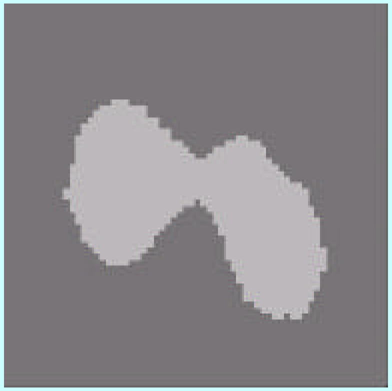


Experiment I: Dynamic Results

True sequence

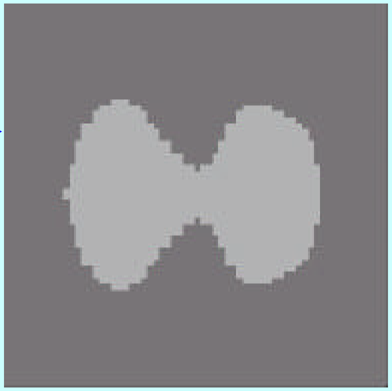


Our reconstruction

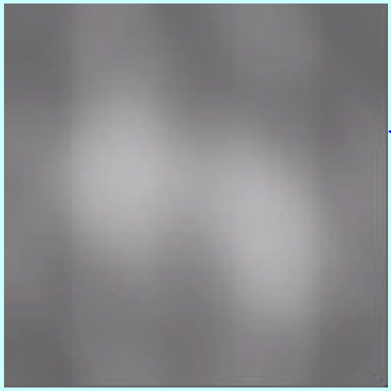


Frame-by-frame recon

Object based, but no temporal smoothness



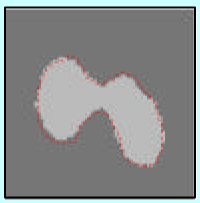
3D Tikhonov recon



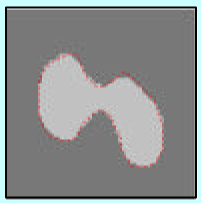
temporally smoothed, but no object model

Experiment I: Static Results

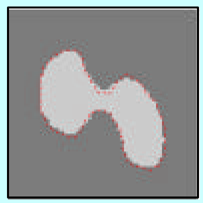
Compared with true boundaries:



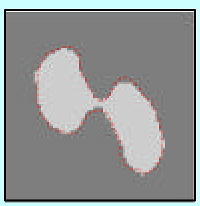
k = 1



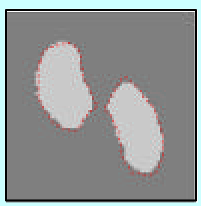
k = 3



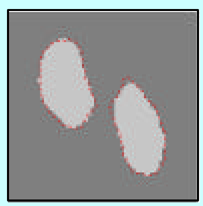
k = 6



k = 9

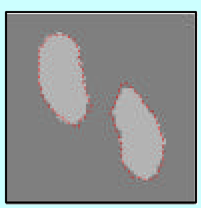


k = 12

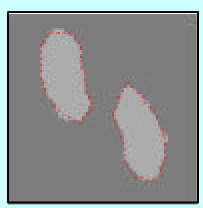


k = 14

k = 16

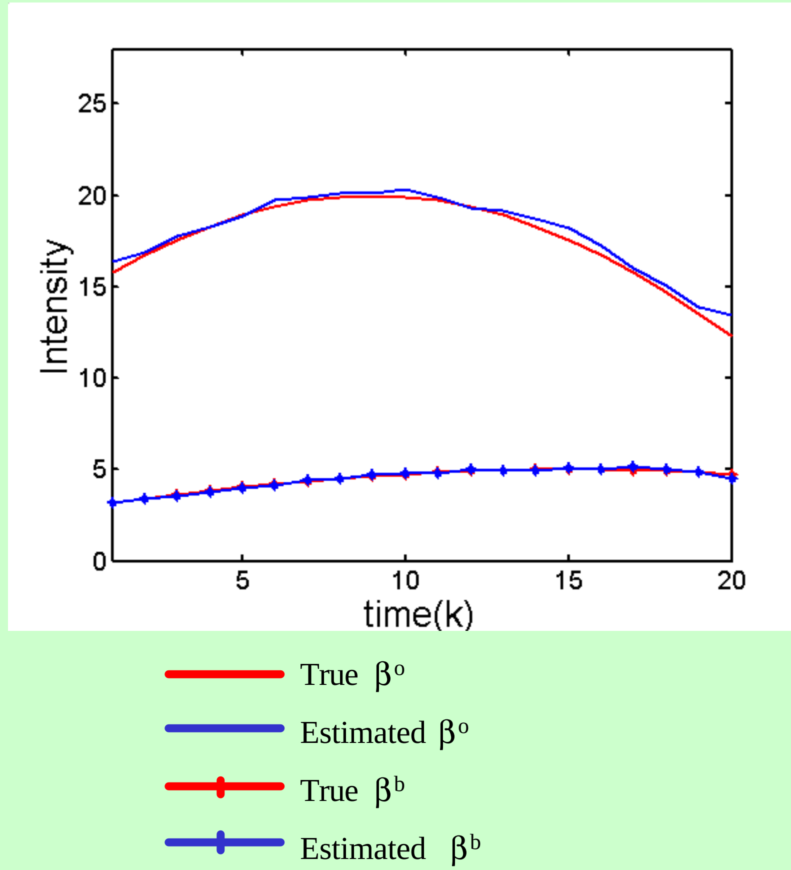


k = 18



k = 20

Time varying intensities



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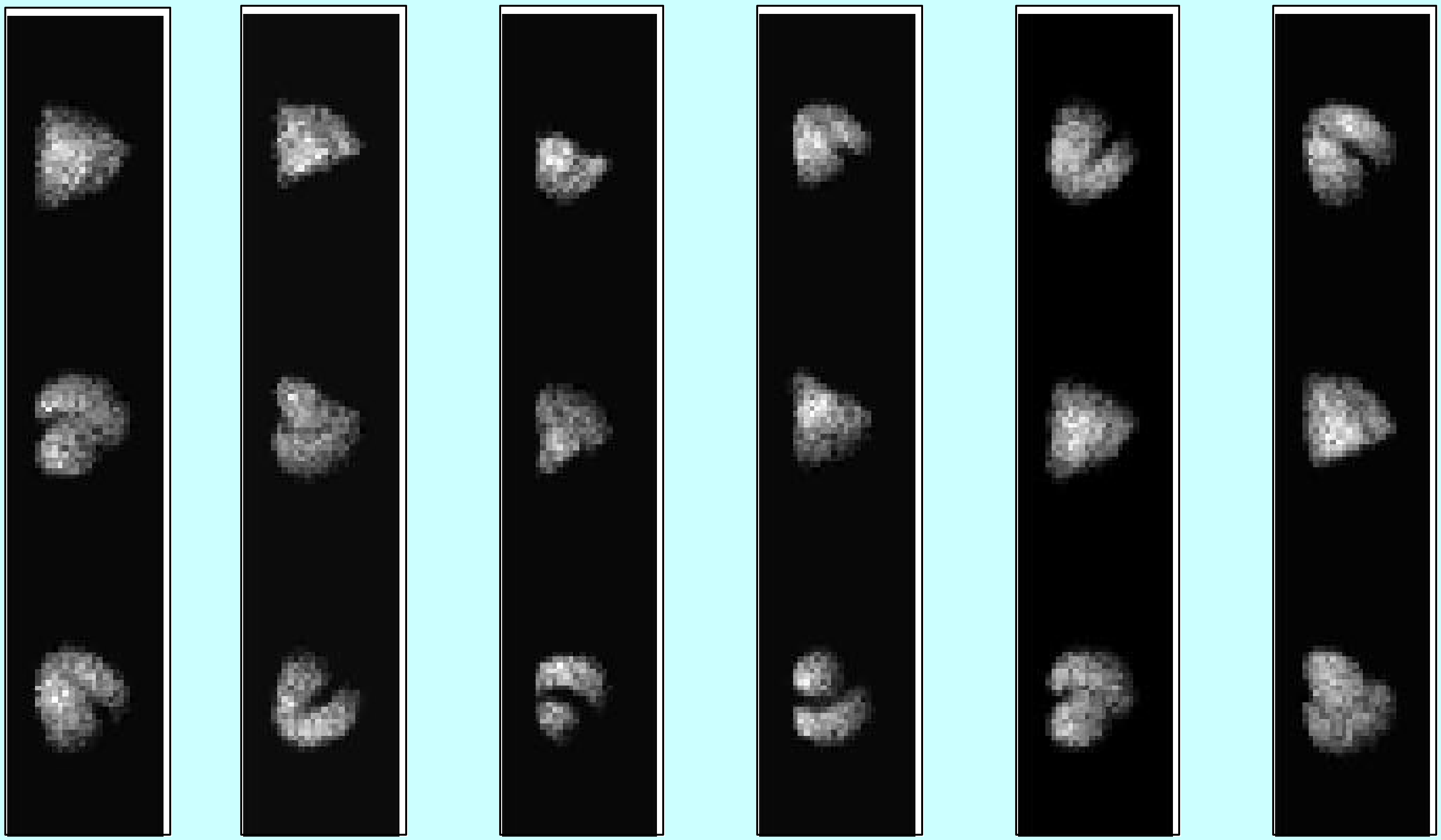
Experiment II:3D + Time

- Experiment setup
  - Beating left and right ventricles from MCAT phantom

P.H.Pretorius, W.Xia, M.A.King, B.M.W.Tsui, T.S.Pan, and B.J.Villegas, " Determination of left and right ventriclular volume and ejection fraction using a mathematical cardiac torso phantom for gated blood pool SPECT", J Nucl Med, 1996.

  - Projection kernel: parallel line integration
  - 3 view angles per time instant
  - View angles change over time
  - Dynamic models A_k,B_k : identity transform
  - Gaussian noise: 15 dB
  - Sequence size : 32*32*28*32

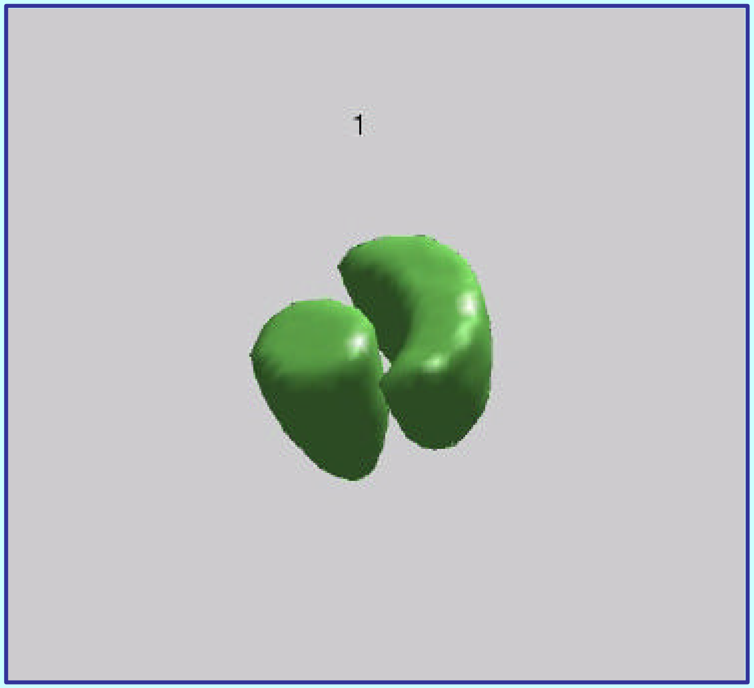
Experiment II: Projection Data



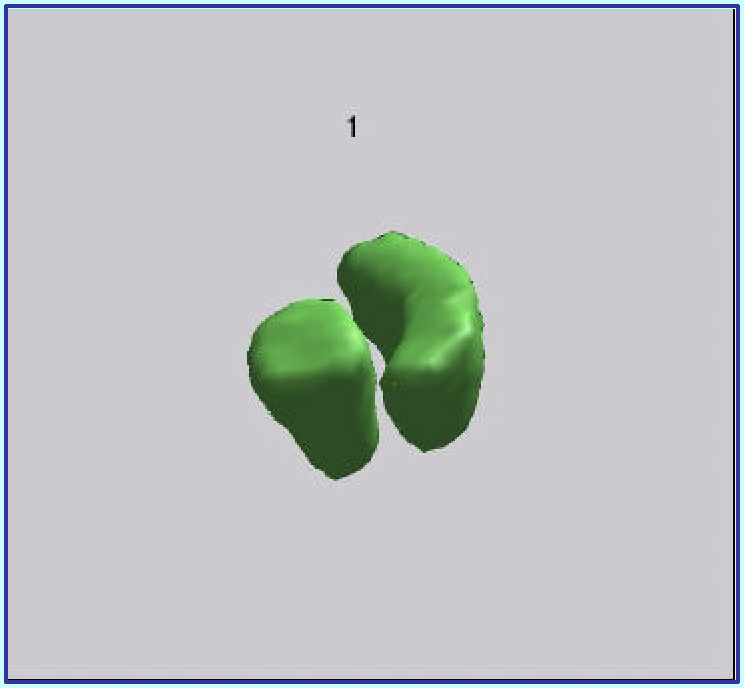
k =1 k =6 k =11 k =16 k =21 k = 26

Experiment II: Results

True sequence

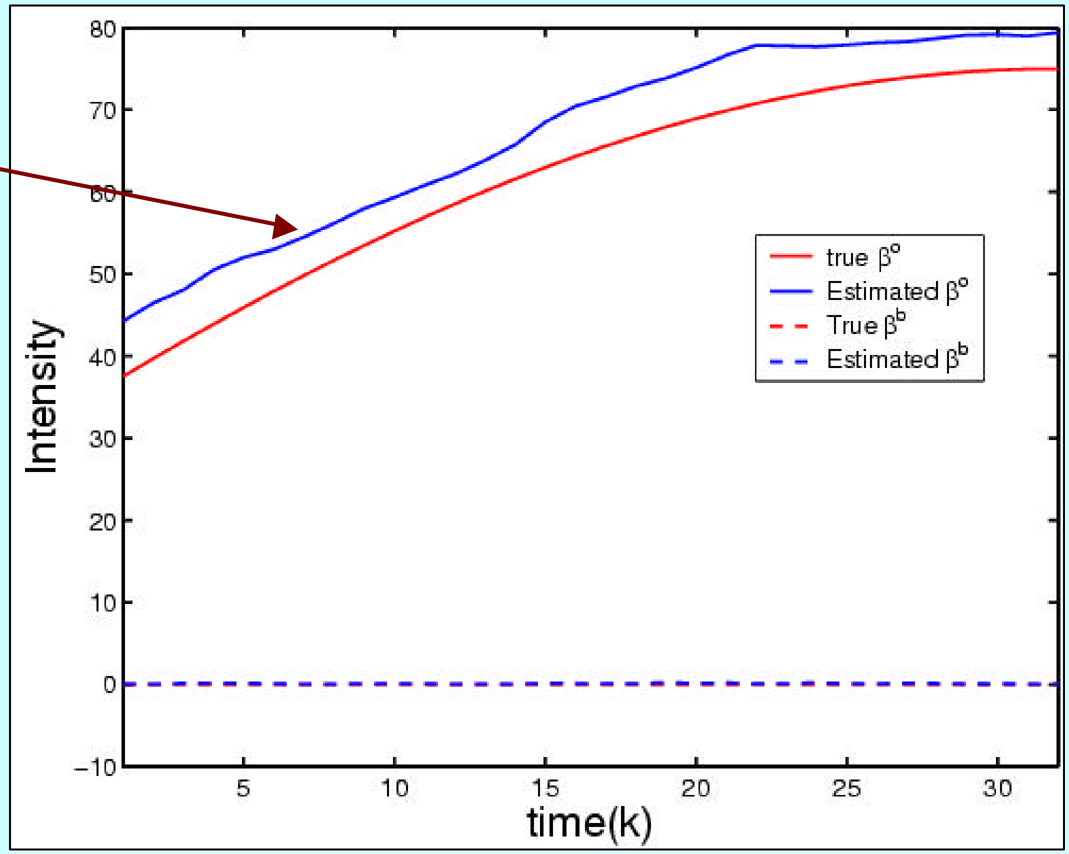


Recon sequence



Experiment II: Intensity Curves

Bias due to regularization



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Extensions

- Up to now:
  - Our approach for object-based dynamic tomography
  - Modeling object dynamics
  - Dynamic shape models assumed known a priori up to now.
- Extensions:
  - Learning shape dynamics
  - Sub-problem: shape matching

Learning Shape Dynamics

- Motivation:
  - A_k maybe unknown in practice.
  - Also of interest for tracking applications
- Problem set up:

Inputs:

M training sequences

$$\left\{\begin{matrix} C_1^1, & C_2^1, & \cdots, & C_K^1 \\ C_1^2, & C_2^2, & \cdots, & C_K^2 \\ \vdots & \vdots & \vdots & \vdots \\ C_1^M, & C_2^M, & \cdots, & C_K^M \end{matrix}\right\}$$

Outputs:

Estimated affine transforms: ***A_k*, *k=1,…,K-1*.**

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Energy-based Method

- Estimate affine transforms as the minimizer of the energy function:

$$J = \underbrace{\sum_{m=1}^{M} \sum_{k=1}^{K-1} W\left(C_{k+1}^{m}, \hat{C}_{k+1}^{m}\right)}_{} + \underbrace{l \sum_{k=1}^{K-1} \left\| \underline{a}_{k+1} - \underline{a}_{k} \right\|^{2}}_{}$$

Data fidelity: distance between shapes

$$\hat{C}_{k+1}^{m} = A_k\left(C_{k+1}^{m}\right)$$

Regularization term

Vector of unknown parameters of A_k

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Sub-problem: Shape Matching

- Given a distance measure, estimate an affine transform minimizing the distance of two curves ***C_1*** and ***C_2***:

$$A(;\underline{a}) = \arg\min_{\underline{a}} W(C_1, \underbrace{A(C_2;\underline{a})})$$

Affine transform

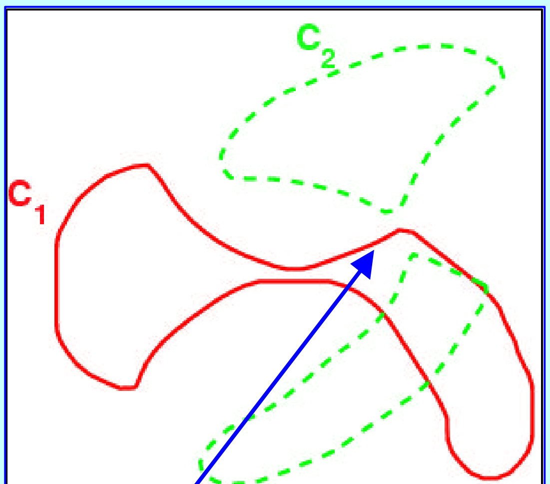
Unknown parameters to be estimated

Distance between two curves

$\hat{C}_2$
Transformed curve

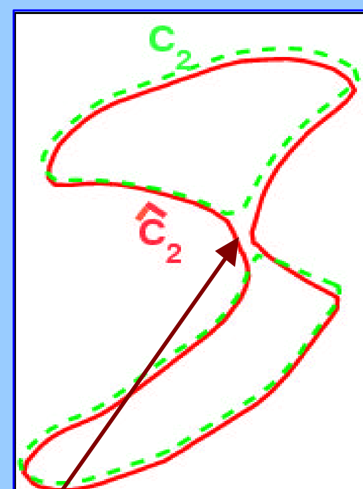
Experiment III: Shape Matching

- Training data



Topological changes

Estimation results



Transformed curve

$$\hat{A}(\underline{x}) = \hat{L}\underline{x} + \hat{b} \quad \text{with}$$

$$\hat{L} = \begin{bmatrix} 0.2460 & 0.9626 \\ -0.8869 & 0.4780 \end{bmatrix}$$

$$\hat{b} = \begin{bmatrix} 6.3356 \\ 4.3744 \end{bmatrix}$$

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Conclusions

- Variational approach for tomographic reconstruction of dynamic objects
- Shape dynamics based on distance between shapes
- Curve evolution and level set methods for implementation
- Extensions to learning shape dynamics and shape matching

Thank You!